C2 Jan 2011 (MA)

Q1a) $\underline{f(1) = 7}$: $1 + 1 + 2 + a + b = 7$

$$a + b = 7 - 4$$

$$\underline{\underline{a + b = 3}}$$

b) $\underline{f(-2) = -8}$: $(-2)^4 + (-2)^3 + 2(-2)^2 - 2a + b = -8$

$$16 - \cancel{8} + 8 - 2a + b = -\cancel{8}$$

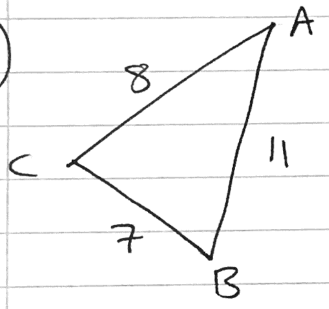
$$\underline{\underline{2a - b = 24}}$$

$$+ \begin{bmatrix} 2a - b = 24 \\ a + b = 3 \end{bmatrix}$$

$$3a = 27$$

$$\therefore \underline{\underline{a = 9}} \qquad \therefore b = 3 - 9 = \underline{\underline{-6}}$$

Q2a)



cosine rule

$$\cos C = \frac{8^2 + 7^2 - 11^2}{2(8)(7)} = -\frac{1}{14}$$

$$\angle ACB = \cos^{-1}\left(-\frac{1}{14}\right) = \boxed{1.64^c}$$

b) $\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2} \times 8 \times 7 \times \sin(1.64) \simeq \boxed{27.9}$ cm^2

Q3a) $\left.\begin{array}{l} ar^4 = -6 \\ ar = 750 \end{array}\right\}$ $\frac{ar^4}{ar} = r^3 = \underline{\underline{\frac{-6}{750}}}$ $\therefore r = \sqrt[3]{\frac{-6}{750}}$

$$= \boxed{-0.2}$$

b) $ar = 750$

$\therefore a = \frac{750}{r} = \frac{750}{-0.2} = \boxed{-3750}$

c) $S_\infty = \frac{a}{1-r} = \frac{-3750}{1--0.2} = \boxed{-3125}$

Q4a) $y = (x+1)(x-5) = 0$

$\underline{\underline{x=-1}}, \ \underline{\underline{x=5}}$

$A(-1, 0) \qquad B(5, 0)$

b) $R = \left| \int_{-1}^{5} (y)\,dx \right| \qquad y = x^2 - 4x - 5$

$\Rightarrow \int_{-1}^{5} [x^2 - 4x - 5]\,dx = \left[\frac{x^3}{3} - 2x^2 - 5x\right]_{-1}^{5}$

$\Rightarrow \left[\frac{125}{3} - 50 - 25\right] - \left[\frac{-1}{3} - 2 + 5\right]$

$\Rightarrow \frac{-100}{3} - \frac{8}{3}$

$= \underline{\underline{-36}} \quad \therefore \quad R = |-36| = \boxed{36}$

(units^2)

Q5a) $_{40}C_4 = {}_nC_r = \dfrac{40!}{(4)!\underbrace{(40-4)}_{\boxed{b=36}}!} = \left[\dfrac{(n)!}{(r)!\,(n-r)!}\right]$

b) $(1+x)^{40} = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \dfrac{n(n-1)(n-2)}{3!}x^3$

$\underbrace{+\dfrac{n(n-1)(n-2)(n-3)}{4!}x^4 + \dfrac{n(n-1)\ldots(n-4)}{5!}x^5}$

$\dfrac{40(39)(38)(37)}{4!}x^4 + \dfrac{40(39)(38)(37)(36)}{5!}x^5$

$\therefore \boxed{p = 91390}$ and $\boxed{q = 658008}$

$\therefore \dfrac{q}{p} = \dfrac{658008}{91390} = \boxed{7.2}$

Q6a)

x	2.5	2.75
y	0.30	0.24

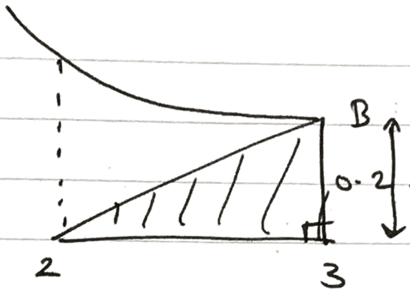
$h = \dfrac{b-a}{n} = \dfrac{3-2}{4} = \dfrac{1}{4}$

b) Area $\simeq \frac{1}{2} \times \frac{1}{4}\left[0.5 + 0.2 + 2(0.38 + 0.30 + 0.24)\right]$

$\simeq \boxed{0.318}$

$\displaystyle\int_2^3 \left[\frac{5}{3x^2-2}\right] dx \simeq 0.318$

c)



Area $\Delta = \frac{1}{2} \times 1 \times 0.2 = 0.1$

$\therefore S = 0.3175 - 0.1 = \boxed{0.2175}$

Q7a) $3\sin^2 x + 7\sin x = 1 - \sin^2 x - 4$

$4\sin^2 x + 7\sin x + 3 = 0$

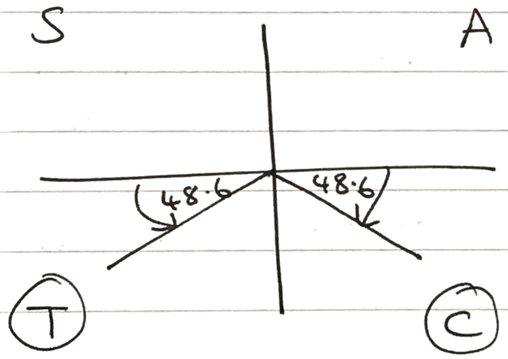
$0 \leq x < 360°$

b) $(4\sin x + 3)(\sin x + 1) = 0$

$4\sin x + 3 = 0$

$\sin x = -\frac{3}{4}$

$x = \sin^{-1}\left(-\frac{3}{4}\right) = -48.59°$



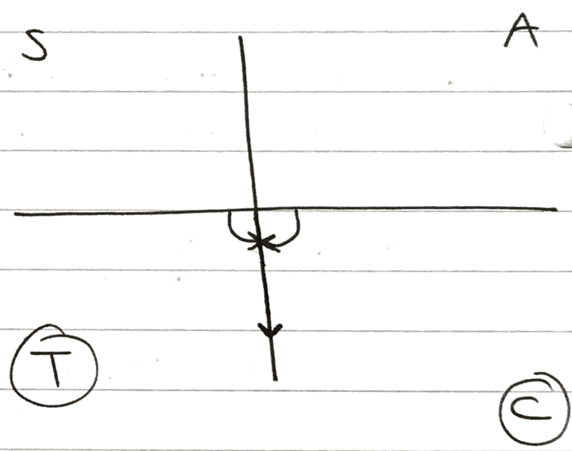
$x = 180 + 48.6,$
$360 - 48.6$

$\boxed{x = 228.6°, 311.4°}$

$\sin x + 1 = 0$

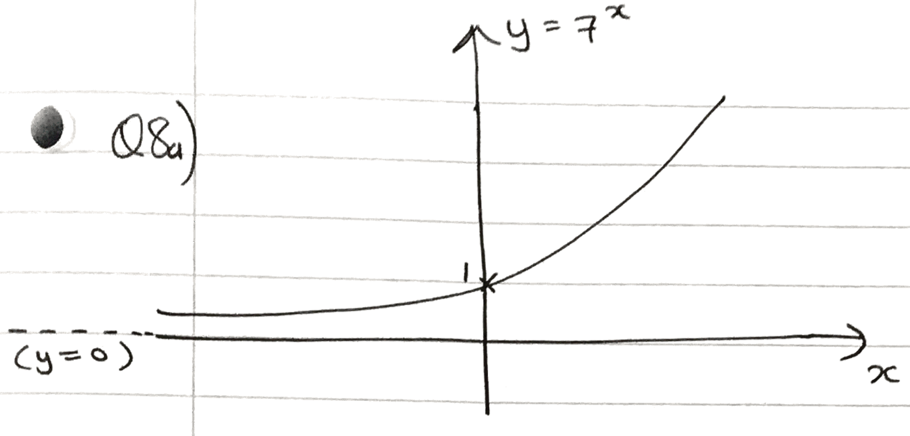
$\sin x = -1$

$x = \sin^{-1}(-1) = -90°$



$\boxed{x = 270°}$

Q8a)



$\underline{x=0}$: $y = 7^0 = 1$.

b) $7^{2x} - 4(7^x) + 3 = 0$

$\underline{\text{let } y = 7^x}$: $y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3 \qquad y = 1$

$7^x = 3$

$\log(7^x) = \log(3)$

$x \log 7 = \log 3$

$x = \dfrac{\log 3}{\log 7} = \boxed{0.56}$

$7^x = 1$

$\hookrightarrow \log(7^x) = \log(1)$

$x \log 7 = 0$

$x = \dfrac{0}{\log 7} = \boxed{0 = x}$

Q9a) midpoint of AB = centre = $\left(\dfrac{-2+8}{2}, \dfrac{11+1}{2}\right)$

$= (3, 6)$

b) length AB = $\sqrt{(8--2)^2 + (1-11)^2} = 10\sqrt{2}$ = diameter

$\therefore$ radius $= 5\sqrt{2}$ //

so $(x-3)^2 + (y-6)^2 = 50$

$[(5\sqrt{2})^2 = 50]$

c) $(x-3)^2 + (y-6)^2 = 50$

$\underline{x=10,\ y=7}$: LHS $= (10-3)^2 + (7-6)^2$

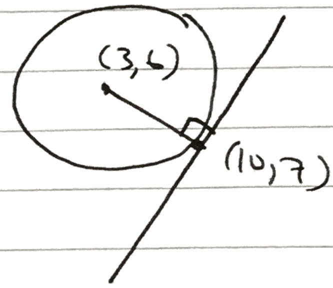
$= 7^2 + 1^2 = 49 + 1 = 50 = \text{RHS}$ //

$\therefore$ (10, 7) lies on C.

d) line joining centre to (10, 7) has gradient $= \dfrac{7-6}{10-3}$

$= \dfrac{1}{7}$ //

$\therefore$ tangent has gradient $= -7$ // $(-7 \times \frac{1}{7} = -1)$



$\therefore y - 7 = -7(x-10)$

$y = -7x + 70 + 7$

$\Rightarrow \boxed{y = -7x + 77}$

Q10a) $V = 4x(25 - 10x + x^2) = 100x - 40x^2 + 4x^3$

$\therefore \dfrac{dV}{dx} = 100 - 80x + 12x^2$

b) $\dfrac{dV}{dx} = 0$: $100 - 80x + 12x^2 = 0$

$(4x - 20)(3x - 5) = 0$

$x = 5$ // $x = 5/3$ //

using $x = \frac{5}{3}$: $V = \frac{4 \times 5}{3}\left(25 - \frac{50}{3} + \frac{25}{9}\right) = \frac{2000}{27} = \boxed{74.1}\ \text{cm}^3$

c) $\dfrac{d^2V}{dx^2} = 24x - 80$

$x = \dfrac{5}{3}$: $\dfrac{d^2V}{dx^2} = \dfrac{24 \times 5}{3} - 80 = -40 < 0$

$\therefore$ This value of V is max.